

# Graphs With Small Intersection Dimension

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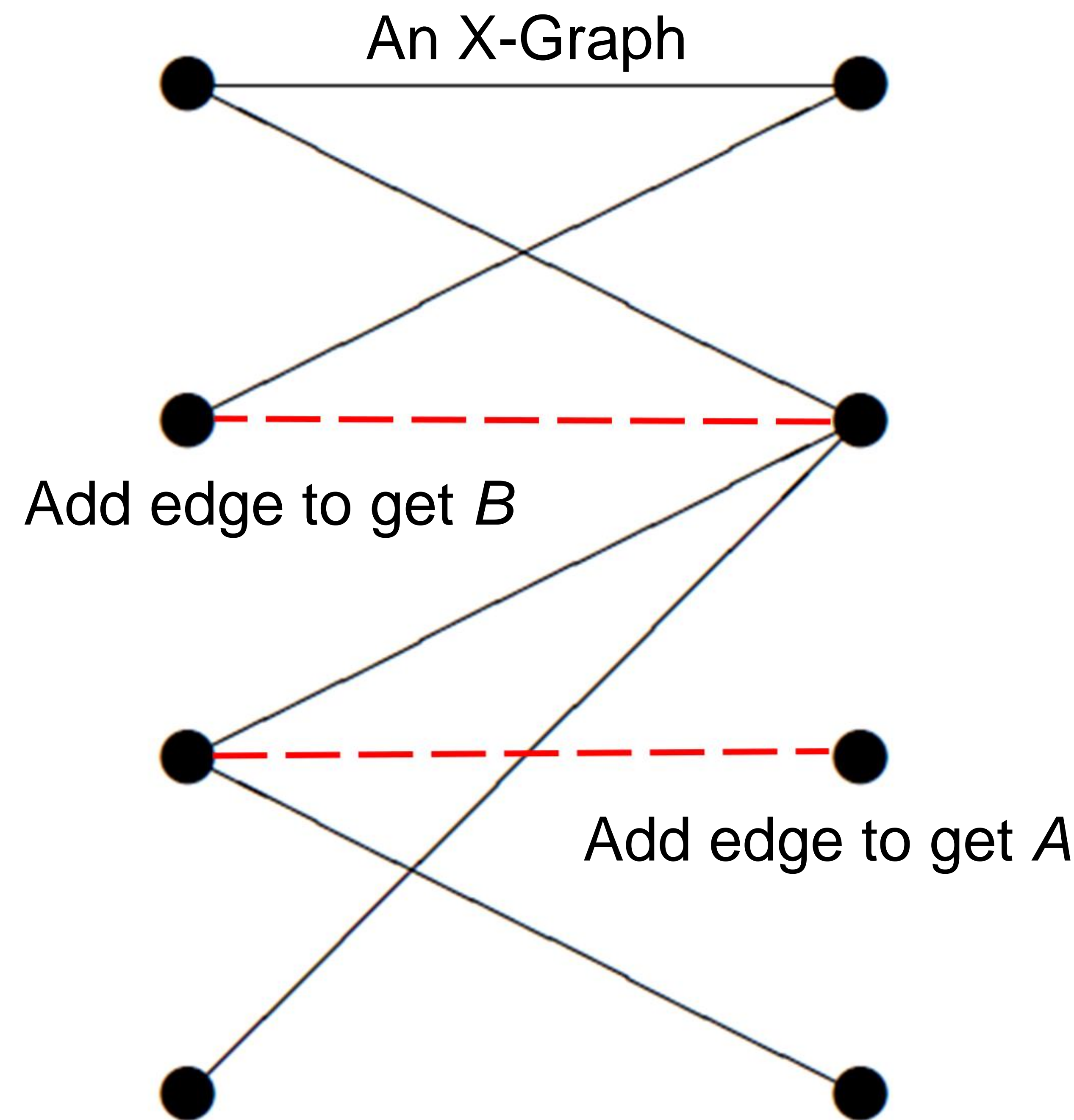
## Abstract

The boxicity of a graph  $G$ , denoted as  $\text{box}(G)$ , is defined as the minimum integer  $k$  such that  $G$  is an intersection graph of axis-parallel  $k$ -dimensional boxes. We examine some known properties of graphs with respect to boxicity, as well as show boxicity results pertaining to several classes of graphs, including split graphs, X-graphs, and powers of trees. We also propose efficient algorithms to produce the relevant  $k$ -dimensional representations.

## Introduction

The graph classes we study all have low bounds on boxicity (e.g. a tree has boxicity at most 2), or some result pertaining to small boxicity (e.g. it is NP-complete to determine if a split graph has boxicity at most 3). We study specific subclasses of these graph classes in pursuit of generalizable results. We examine those split graphs formed from various classes of bipartite graphs by turning one part into a clique. Specifically, we find results for split graphs of convex graphs and X-graphs.

A recent result gave a bound of  $k+1$  on the boxicity of the  $k^{\text{th}}$  power of a tree. We improve this bound for trees with maximum degree ( $\Delta$ ) bounded at 3.



## Powers of Trees

We find a constant bound on the boxicity of powers of trees with  $\Delta$  at most 3; any even power of such a tree has boxicity at most 4, any odd power has boxicity at most 5. This result is shown by first embedding the tree  $T$  in a revised perfect binary tree  $T'$  (see right), and then using the symmetrical structure of such trees to construct pairs of interval graphs whose intersection is  $T'$ . This gives us a result on the boxicity of powers of  $T'$ , which in turn can be applied to the original tree  $T$ .

## Split Graphs

We prove that the split graph of a convex graph has boxicity at most 2, using intersecting chain graphs. A chain graph is always an interval graph, so a 2 chain graph representation is equivalent to a 2-dimensional box representation. We then prove that any X-Graph is the intersection of 2 convex graphs,  $A$  and  $B$  (see left). As any convex graph has boxicity at most 2, any X-graph then has boxicity at most 4.

